Loughborough University

Developing university mathematics teaching through collaborative inquiry: teacher-researchers working and learning

TSUKUBA -- SEAMEO -- 2016

Collaborative teaching-research team

3 experienced teachers working with a mathematics module for first year engineering students

- 2 with long experience of teaching mathematics to engineering students: roles – design, discuss, advise, analyse data
- 1 with long experience of mathematics teaching through inquiry approaches: roles – design, teach, reflect, feed back, analyse data

1 educational researcher

- With experience of working in university mathematics education: roles – collect data, stimulate reflection and feedback, analyse data

Innovation in teaching mathematics to engineering students (ESUM)

- Seeking to promote engineering students’ more conceptual understandings in a first year mathematics module

- Researcher observes and records all lectures and tutorials, surveys students and conducts post module focus group interviews

Teaching team designs tasks around mathematical topics to encourage student engagement and inquiry in mathematics

For example, a task relating to functions with a focus on linear/quadratic relationships

Exploring functions and equations – an inquiry-based task

1a) Consider the function

$$f(x) = x^2 + 2x \quad (x \text{ is real})$$

Give an equation of a line that intersects the graph of this function:

Twice; Once; Never.

1b) If we have the function

$$f(x) = ax^2 + bx + c.$$

What can you say about lines which intersect this function twice

1c) Write down equations for three straight lines and draw them in GeoGebra

Find a (quadratic) function such that the graph of the function cuts one of your lines *twice*, one of them *only once*, and the third *not at all* and show the result in GeoGebra.

Repeat for three *different* lines

(what does it mean to be different?)

The innovation and its goals (the micro)

4 new elements

- Inquiry-based teaching/learning
- Geogebra environment for work on functions
- Small group activity in tutorials
- Small group project – assessed

Jaworski & Matthews (2011)
Jaworski, Robinson, Matthews and Croft (2012)

Goals

To engage students in/with mathematics

To encourage linking of different representations

To facilitate dialogue, interchange and sharing of ideas

To motivate activity

Jaworski (2015)

Macro factors

- 2-semester module – 2 teachers – formal exam at end (innovation in first semester only -- 2 lectures & 1 tutorial per week for 13 weeks)
- Given content specification (pre-calculus, calculus, vectors & matrices …)
- Designated lectures and tutorials in traditional lecture theatres and seminar rooms
- Students come from school learning/teaching experiences
- Students' wide range of mathematical experience (some don't have A levels in mathematics)
- First year grades do not count to the degree

The Sociocultural Scene

- Considering the micro AND the macro
  - Micro: teaching/learning activity, interactions and insights
  - Macro: wider influences impacting on the micro
- Sociocultural focus links teaching goals and classroom interactions with institutional, systemic and cultural influences:
- e.g.,
  - considering the nature of the setting and how factors involved in the academic infrastructure affect teaching/learning activity
  - looking at the cultural underpinnings of perspectives and actions

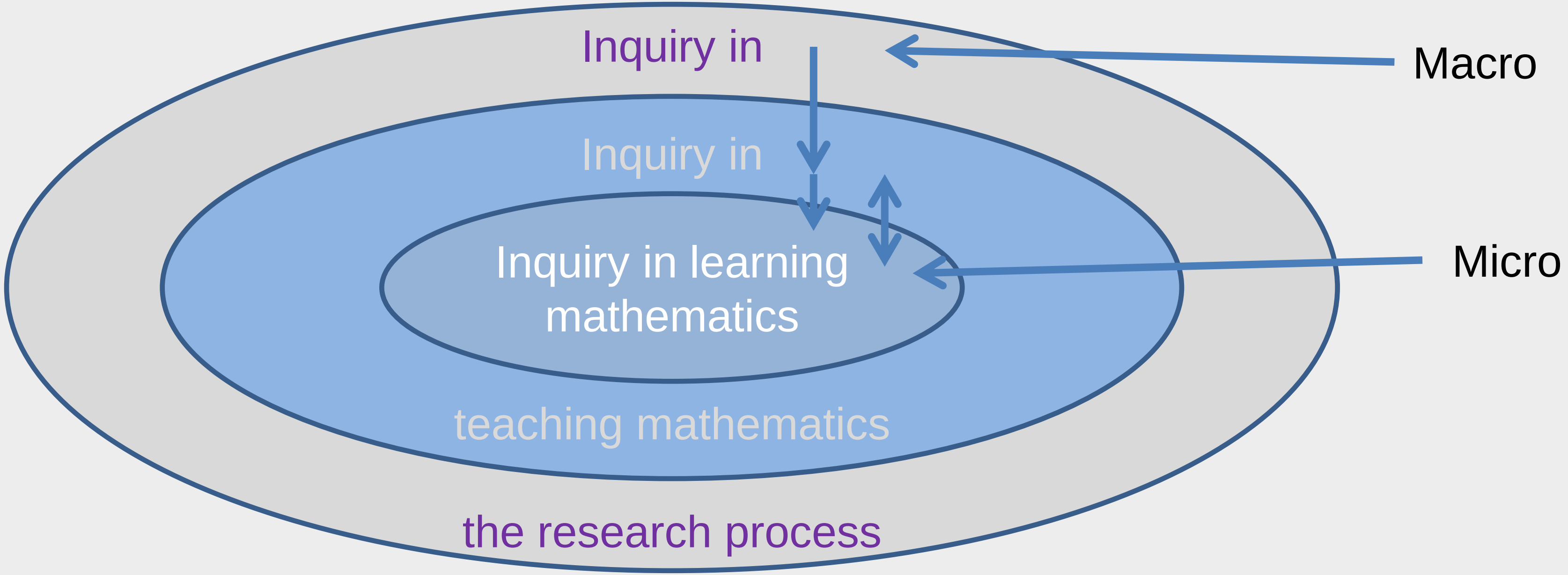
Developmental research: knowledge creation

- Research which promotes development as well as charting or evaluating it. Fundamentally an *inquiry* process
- Teaching-research team – designing, teaching, analysing data, evaluating – *insider* researcher ***– knowledge in practice***
- **Action cycle:**

Plan → Act & Observe → Reflect & Analyse → Feedback

- Researcher – observing, surveying, interviewing, analysing data – *outsider* researcher ***– abstracted knowledge***

Inquiry communities -- three layers of inquiry



Outcomes from the ESUM Project

- Centrality of questioning
- Inquiry-based questions (tutorials)
- Use of GeoGebra
- Small group activity
- Small group projects
- Tests and Exam
- Student perspectives

- Students responding in lectures
- Variety of group responses
- Variety of responses
- Variety of responses
- Generally well done (Web PA)
- Average score 10% higher than previous
- Evidence of conflict

Student Perspectives

We feel being able to explore functions as a group has helped our learning about functions as we can discover together and ask each other questions about how they work and what they can be useful for, and where one student questions something, the whole group benefits from their answer.

Understanding maths – that was the point of GeoGebra wasn't it? Just because I understand maths better doesn't mean I'll do better in the exam. I have done less past paper practice.

I found GeoGebra almost detrimental because it is akin to getting the question and then looking at the answer in the back of the book. I find I can understand the graph better if I take some values for x and some values for y, plot it, work it out then I understand it … if you just type in some numbers and get a graph then you don't really see where it came from.

Learning from outcomes of research

- Feeling pleased with what went well
- Feeding back into ongoing teaching and future teaching from observations and issues arising
- Asking questions about how to improve on what did not go well
- Recognising the big issues in differing perspectives between teaching team and student cohort (micro and macro)
- Seeking ways of addressing these differences in perspective

Activity Theory -- Leont'ev's three levels

activity<->motive, actions<->goals, operations<->conditions

Level	Teaching Team	Students
1	*Activity* is mathematics teaching-learning. For the teacher(s) it is *motivated* by the desire for students to gain a deep conceptual-relational understanding of mathematics. We might in this case call it "teaching-for-learning"	For students the *activity* is learning within the teaching environment and with respect to many external factors (youth culture, school-based expectations of university etc.) and is (probably) *motivated* by the desire to get a degree in the most student-effective way possible.
2	Here, *actions* are design of tasks and inquiry-based questions – with *goals* of student engagement, exploration and getting beyond a superficial and/or instrumental view of mathematics. *Actions* include use of GeoGebra with the *goal* of providing an alternative environment for representation of functions offering ways of visualizing functions and gaining insights into function properties and relationships. *Actions* include forming students into small groups and setting group tasks with the *goals* to provide opportunity for sharing of ideas, learning from each other and articulating mathematical ideas	For students, *actions* involve taking part in the module: attending lectures & tutorials; using the LEARN page; using the HELM books; etc. with *goals* related to student epistemology. So *goals* might include attending lectures & tutorials because this is where you are offered what you need to pass the module; clear views on what ought to be on offer and what you expect from your participation; wanting to know what to do and how to do it; wanting to do the minimum amount of work to succeed; wanting to understand; wanting to pass the year's work.
3	Here we see operations such as the kinds of interactions used in lectures to get students to engage and respond, the ways in which questions are used, the operation of group work in tutorials and interactions between teachers and students. The conditions include all the factors of the university environment that condition and constrain what is possible – for example, if some tutorials need to be in a computer lab, then they all have	*Operations* include degrees of participation – listening in a lecture, talking with other students about mathematics, reading a HELM book to understand some bit of mathematics, using the LEARN page to access a lecture, Powerpoint etc. The conditions in which this fitting in pieces of cour given deadlines, balancing the academic and the social, getting

Activity Theory analysis – Level 2 – Actions ←→ Goals

Teaching Team

For teaching team, *actions* are design of tasks and inquiry-based questions – with *goals* of student engagement, exploration and getting beyond a superficial and/or instrumental view of mathematics. *Actions* include use of GeoGebra with the *goal* of providing an alternative environment for representation of functions offering ways of visualizing functions and gaining insights into function properties and relationships. *Actions* include forming students into small groups and setting group tasks with the *goals* to provide opportunity for sharing of ideas, learning from each other and articulating mathematical ideas

Students

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Emerging challenges

- Recognising the differences in goals, arising from differing orientations and cultures, how can we take into account the macro issues as we plan at the micro level?
- One response is to bring students into the planning, so that they gain insights into teaching goals and can respond, as insiders, from their own perspectives and cultures – requires explicit effort to overcome power differentials.
- This was achieved in the SYMBoL project – another story!

Thank You ☺

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